

Exercise 67

Find the equation of the line that passes through the following points: $(a, 0)$ and (c, d)

Solution

The general equation for a line is

$$Y = MX + B.$$

The first point says that when the input is $X = a$, the output is $Y = 0$.

$$0 = M(a) + B$$

The second point says that when the input is $X = c$, the output is $Y = d$.

$$d = M(c) + B$$

This is a system of two equations for two unknowns that can be solved for.

$$\begin{cases} aM + B = 0 \\ cM + B = d \end{cases}$$

Subtract the respective sides of these equations to eliminate B .

$$aM - cM = 0 - d \quad \rightarrow \quad (a - c)M = -d \quad \rightarrow \quad M = \frac{-d}{a - c}$$

Multiply both sides of the first equation by $1/a$, multiply both sides of the second equation by $-1/c$,

$$\begin{cases} M + \frac{B}{a} = 0 \\ -M - \frac{B}{c} = -\frac{d}{c} \end{cases}$$

and add the respective sides to eliminate M .

$$\frac{B}{a} + \left(-\frac{B}{c}\right) = 0 + \left(-\frac{d}{c}\right) \quad \rightarrow \quad \left(\frac{1}{a} - \frac{1}{c}\right)B = -\frac{d}{c} \quad \rightarrow \quad \frac{c - a}{ac}B = -\frac{d}{c} \quad \rightarrow \quad B = -\frac{ad}{c - a}$$

Now that M and B are solved for, the equation of the line is known.

$$Y = \frac{-d}{a - c}X - \frac{ad}{c - a}$$